Answers for 5.6

5.6 Skill Practice

1. You temporarily assume that the desired conclusion is false and this leads to a logical contradiction.

2. *Sample answer:* Consider opening a door; the wider you open it, the bigger the angle the door makes with the wall and the larger the opening.

3. >

4. <

5. <

6. <

7. =

8. >

9. A

10. $m\angle DGE < m\angle FGE$; Converse of the Hinge Theorem applies.

11. Suppose xy is even.

12. Suppose $\angle B$ is a right angle.

13. $\angle A$ could be a right angle or a straight angle.

14. To use the Hinge Theorem the angle must be the included angle between the two pairs of congruent sides.

15. The Hinge Theorem is about triangles not quadrilaterals.

16. $x < 30\frac{1}{2}$

17. $x > \frac{1}{2}$

18. $x > \frac{3}{2}$

19. Using the Converse of the Hinge Theorem $\angle NRQ > \angle NRP$. Since $\angle NRQ$ and $\angle NRP$ are a linear pair $\angle NRQ$ must be obtuse and $\angle NRP$ must be acute.

20. In $\triangle FHG$, $m\angle H > m\angle GFH$ or $m\angle FGH$ by Theorem 5.11.

21. $\triangle ABC$ is obtuse; the orthocenter of an obtuse triangle is always outside of the triangle, making the angle smaller outside than inside.

5.6 Problem Solving

22. the first hiker; the Hinge Theorem

23. E, A, D, B, C

24. Assume temporarily that either $EF < DF$ or $EF = DF$.
Case 1: Assume $EF < DF$. Using Theorem 5.10 you would get $m\angle D < m\angle E$ which contradicts the given statement. Case 2: Assume $EF = DF$. If $EF = DF$ then $\triangle FDE$ would be isosceles with $m\angle D = m\angle E$ which contradicts the given statement. Therefore, the assumption must be false. So, $EF > DF$.

25. a. It gets larger; it gets smaller.

b. *KM*

c. *Sample answer:* Since $NL = NK = NM$ and as $m\angle LNK$ increases KL increases and $m\angle KNM$ decreases, as KM decreases you have two pairs of congruent sides with $m\angle LNK$ eventually greater than $m\angle KNM$. The Hinge Theorem guarantees KL will eventually be greater than KM.

26. *Sample answer:* Assume temporarily that $\overline{AB}$ is not the shortest distance from A to k. This implies that there is a point C on k such that $\overline{AC}$ is the shortest distance. $\triangle ABC$ is a right triangle with hypotenuse $\overline{AC}$. Since $\overline{AC}$ is opposite the right angle, the largest angle in a right triangle, Theorem 5.10 guarantees that $\overline{AC}$ is the largest side. This contradicts the assumption that $\overline{AC}$ is the shortest side, making $\overline{AB}$ the shortest side.

27. Prove: If x is divisible by 4, then x is even. Proof: Since x is divisible by 4, $x = 4a$. When you factor out a 2, you get $x = 2(2a)$ which is in the form $2n$, which implies x is an even number; you start the same way by assuming what you are to prove is false, then proceed to show this leads to a contradiction.

28. *Sample answer:* Step 1: Since $\overline{BC} \cong \overline{EF}$, $\angle CBP \cong \angle FED$ by construction, and $\overline{BP} \cong \overline{ED}$ by construction, you have $\triangle PBC \cong \triangle DEF$ by SAS. Step 2: Since $\overrightarrow{BH}$ bisects $\angle PBA$ by construction, you have $\angle PBH \cong \angle ABH$. Using Transitive Property of Segment Congruence, you know that $\overline{AB} \cong \overline{PB}$. Using the Reflexive Property of Segment Congruence, you have $\overline{BH} \cong \overline{BH}$. Therefore $\triangle ABH \cong \triangle PBH$ by SAS. Step 3: $AC = AH + HC$ by the Segment Addition Postulate. $AH + HC = PH + HC$ by corr. parts of $\cong$ ⚠ are $\cong$. $PH + HC > PC$ by the Triangle Inequality Theorem. $PC = DF$ by corr. parts of $\cong$ ⚠ are $\cong$.

5.6 Mixed Review

29. $\dfrac{1\ \text{ft}}{12\ \text{in.}}$ **30.** $\dfrac{1\ \text{kL}}{1000\ \text{L}}$ **31.** $\dfrac{16\ \text{oz}}{1\ \text{lb}}$

32. <u>Equation (Reason)</u>

$1.5(x + 4) = 5(2.4)$ (Given)

$1.5(x + 4) = 12$ (Simplify.)

$x + 4 = 8$ (Division Property of Equality)

$x = 4$ (Subtraction Property of Equality)

33. <u>Equation (Reason)</u>

$-3(-2x + 5) = 12$ (Given)

$-2x + 5 = -4$ (Division Property of Equality)

$-2x = 9$ (Subtraction Property of Equality)

$x = \dfrac{9}{2}$ (Division Property of Equality)

34. <u>Equation (Reason)</u>

$2(5x) = 3(4x + 6)$ (Given)

$10x = 3(4x + 6)$ (Simplify.)

$10x = 12x + 18$ (Distributive Property)

$-2x = 18$ (Subtraction Property of Equality)

$x = -9$ (Division Property of Equality)

35. $-\dfrac{2y}{7x}$

5.4–5.6 Mixed Review of Problem Solving

1. a. no

  b. yes

  c. altitude and median

2. Dawson. *Sample answer:* The Hinge Theorem guarantees that Allentown to Dawson is the shortest distance since the included angle is 120°, unlike Allentown to Bakersville where the included angle is 145°.

3. 18;

4. No. *Sample answer:* In a triangle the largest angle should be opposite the longest side. The side measuring 13.55 centimeters is opposite the right angle yet 13.7 centimeters is the longest side in the right triangle on the left.

5. a. 8 ft $< \ell <$ 40 ft

   b. 16 ft, 24 ft, 32 ft

   c. 24 ft by 16 ft by 32 ft; since two of the sides are 24 feet and 16 feet, the third side must be 32 feet and so the dog can run at least 25 feet within the pen.

6. 0.63 mm $< DE <$ 1.03 mm